تمرين 1 : احسب النهايات التالية:

$$\lim_{x \mapsto +\infty} \sqrt{\sqrt{x}+1} - \sqrt{x+\sqrt{x}} \qquad \lim_{x \mapsto -\infty} x+3-\sqrt{x^2+4x} \qquad \lim_{x \mapsto +\infty} x+3-\sqrt{x+3}$$

$$\lim_{x \mapsto -\infty} \left(\sqrt{5x^2+x+1}-mx\right) \quad حيث \quad m \in IR \qquad \lim_{x \mapsto +\infty} \sqrt{1+2x^3}-\sqrt{x^3+x+1}$$

$$\lim_{x \mapsto +\infty} \frac{\cos(x)+5\sin(x^2)}{x^4+x^2+1} \qquad \lim_{x \mapsto -\infty} \left(\frac{x+1-\sqrt{1-x}}{x^2-\sqrt{x^2+2}}\right) \qquad \lim_{x \mapsto +\infty} \left(\frac{x-\sqrt{x^2+1}}{x^2-\sqrt{x^4+1}}\right)$$

تمرين 2 : احسب النهايات التالية:

$$\lim_{x \mapsto -2^-} \left(\frac{\sqrt{x^2-4}}{x+2}\right) \qquad \lim_{x \mapsto 2^+} \left(\frac{\sqrt{x^2-4}}{x-2}\right) \qquad \lim_{x \mapsto -1} \left(\frac{\sqrt{1-3x}-2}{x+1}\right)$$

$$\lim_{x \mapsto 0^+} \left(\frac{1}{x}+\cos\left(\frac{1}{x}\right)\right) \qquad \lim_{x \mapsto 1} \left(\sin\left(\frac{5}{1-x^3}\right)(x^2-2x+1)\right) \qquad \lim_{x \mapsto 1} \left(\frac{x^2+x-2}{x^3+4x^2-8x+3}\right)$$

تمرين 3 : احسب النهايات التالية:

$$\lim_{x \mapsto 0} \left(\frac{\sqrt{3}\cos x-\sin^2 x-\sqrt{3}}{x^2}\right) \qquad \lim_{x \mapsto 3} \frac{\sin(\pi x)}{x-3} \qquad \lim_{x \mapsto 0} \left(\frac{1}{\sqrt{3}\cos x-\sin^2 x-\sqrt{3}}\right)$$

$$\lim_{x \mapsto \frac{\pi}{4}} \left(\frac{\sqrt{1-\cos(x)}-\sqrt{1-\sin(x)}}{1-\tan(x)}\right) \qquad \lim_{x \mapsto \frac{\pi}{4}} \left(\frac{\tan x-1}{2\cos x-\sqrt{2}}\right) \qquad \lim_{x \mapsto \frac{\pi}{6}} \left(\frac{\cos x-\sqrt{3}\sin x}{6x-\pi}\right)$$

تمرين 4 : ليكن $a \in IR$ و f_a الدالة العددية المعرفة بما يلي : $$f_a(x)=\frac{1}{x+a}-\frac{a^2 x^2}{x^3+a^3}$$

1) حدد مجموعة تعريف الدالة f_a

2) حدد قيم العدد a التي من أجلها تقبل f_a نهاية في $-a$

تمرين 5 : ليكن m عددا حقيقيا ولتكن f الدالة العددية المعرفة بما يلي :
$$f(x)=\frac{mx^3+(m-2)x^2+(m-1)x+m-3}{x(x-2)(x-3)}$$

1) حدد مجموعة تعريف الدالة f

2) ادرس نهاية الدالة f عند $+\infty$ و $-\infty$ و 0 حسب قيم البارامتر m

تمرين 6 : لتكن f الدالة العددية المعرفة بما يلي : $$f(x)=\frac{x^2\sqrt{x+2}-8}{4-x^2}$$

1) حدد مجموعة تعريف الدالة f

2) أحسب $\lim_{x \mapsto +\infty} f(x)$

3) بين أن f تقبل تمديدا بالاتصال في 2 وحدده.

تمرين 1 :

$$\lim_{x \mapsto +\infty} x+3-\sqrt{x+3} = \lim_{x \mapsto +\infty} \left(\sqrt{x+3}\right)^2 - \sqrt{x+3} = \lim_{x \mapsto +\infty} \sqrt{x+3}\left(\sqrt{x+3}-1\right) = +\infty$$

$$\lim_{x \mapsto +\infty} x+3-\sqrt{x^2+4x} = \lim_{x \mapsto +\infty} x+2-\sqrt{x^2+4x}+1 = \lim_{x \mapsto +\infty} \frac{x^2+4x+4-x^2-4x}{x+2+\sqrt{x^2+4x}}+1$$

$$= \lim_{x \mapsto +\infty} \frac{4}{x+2+\sqrt{x^2+4x}}+1 = 0+1 = 1$$

$$\lim_{x \mapsto +\infty} \sqrt{\sqrt{x}+1} - \sqrt{x+\sqrt{x}} = \lim_{x \mapsto +\infty} \sqrt{\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)} - \sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)}$$

$$= \lim_{x \mapsto +\infty} \sqrt{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}} - \sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}} = \lim_{x \mapsto +\infty} \sqrt{x}\left(\frac{\sqrt{\sqrt{x}}}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}} - \sqrt{1+\frac{1}{\sqrt{x}}}\right)$$

$$= \lim_{x \mapsto +\infty} \sqrt{x}\left(\frac{1}{\sqrt{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}} - \sqrt{1+\frac{1}{\sqrt{x}}}\right) = -\infty$$

$$\lim_{x \mapsto +\infty} \sqrt{1+2x^3} - \sqrt{x^3+x+1} = \lim_{x \mapsto +\infty} \sqrt{x^3}\sqrt{\frac{1}{x^3}+2} - \sqrt{x^3}\sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}}$$

$$= \lim_{x \mapsto +\infty} \sqrt{x^3}\left(\sqrt{\frac{1}{x^3}+2} - \sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}}\right) = +\infty$$

- إذا كان $m>0$ فإن : $\lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}-mx\right) = +\infty$ $\quad\left(\lim_{x \mapsto -\infty} -mx = +\infty\right)$

- إذا كان $m=0$ ، فإن : $\lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}-mx\right) = \lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}\right) + \infty$

- إذا كان $-\sqrt{5}<m<0$ فإن :

$$\lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}-mx\right) = \lim_{x \mapsto -\infty}\left(\sqrt{x^2\left(5+\frac{1}{x}+\frac{1}{x^2}\right)}-mx\right) = \lim_{x \mapsto -\infty}\left(-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-mx\right)$$

$$= \lim_{x \mapsto -\infty} -x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right) = +\infty$$

- إذا كان $m=-\sqrt{5}$ فإن :

$$\lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}-mx\right) = \lim_{x \mapsto -\infty} \sqrt{5x^2+x+1}+\sqrt{5}\,x = \lim_{x \mapsto -\infty} \frac{x+1}{\sqrt{5x^2+x+1}-\sqrt{5}\,x}$$

$$= \lim_{x \mapsto -\infty} \frac{x\left(1+\frac{1}{x}\right)}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}\,x} = \lim_{x \mapsto -\infty} \frac{1+\frac{1}{x}}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}\,x} = \frac{-1}{2\sqrt{5}}$$

- إذا كان $m<-\sqrt{5}$ فإن : $\lim_{x \mapsto -\infty}\left(\sqrt{5x^2+x+1}-mx\right) = \lim_{x \mapsto -\infty} -x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right) = -\infty$

$$\lim_{x \mapsto +\infty} \left(\frac{x - \sqrt{x^2+1}}{x^2 - \sqrt{x^4-1}} \right) = \lim_{x \mapsto +\infty} \frac{\left(x^2 - (x^2+1)\right)\left(x^2 + \sqrt{x^4-1}\right)}{\left(x^4 - (x^4-1)\right)\left(x + \sqrt{x^2+1}\right)} = \lim_{x \mapsto +\infty} \frac{-\left(x^2 + x^2\sqrt{1 - \frac{1}{x^4}}\right)}{\left(x + x\sqrt{1 + \frac{1}{x^2}}\right)}$$

$$\lim_{x \mapsto +\infty} \left(\frac{x - \sqrt{x^2+1}}{x^2 - \sqrt{x^4-1}} \right) == \lim_{x \mapsto +\infty} \frac{-x\left(1 + \sqrt{1 - \frac{1}{x^4}}\right)}{1 + \sqrt{1 + \frac{1}{x^2}}} = -\infty$$

$$\lim_{x \mapsto -\infty} \left(\frac{x+1 - \sqrt{1-x}}{x^2 - \sqrt{x^2+2}} \right) = \lim_{x \mapsto -\infty} \frac{x\left(1 + \frac{1}{x} - \frac{\sqrt{1-x}}{x}\right)}{x^2 + x\sqrt{1 + \frac{2}{x^2}}} = \lim_{x \mapsto -\infty} \frac{1 + \frac{1}{x} + \sqrt{\frac{1-x}{x^2}}}{x\left(1 + \frac{1}{x}\sqrt{1 + \frac{2}{x^2}}\right)} = 0$$

لدينا : $\forall x \in IR \quad \left| \cos x + 5\sin x^2 \right| \leq \left| \cos x \right| + \left| 5\sin x^2 \right| \leq 1 + 5 \leq 6$

منه : $\forall x \in IR \quad \left| \frac{\cos x + 5\sin x^2}{x^4 + x^2 + 1} \right| \leq \frac{6}{x^4 + x^2 + 1}$ ، وبما أن $\lim_{x \mapsto +\infty} \frac{1}{x^4 + x^2 + 1} = 0$ فإن:

$$\lim_{x \mapsto +\infty} \frac{\cos x + 5\sin x^2}{x^4 + x^2 + 1} = 0$$

تمرين2 :

$$\lim_{x \mapsto -1} \left(\frac{\sqrt{1-3x} - 2}{x+1} \right) = \lim_{x \mapsto -1} \frac{1 - 3x - 4}{(x+1)\left(\sqrt{1-3x}+2\right)} = \lim_{x \mapsto -1} \frac{-3}{\sqrt{1-3x}+2} = \frac{-3}{4}$$

$$\lim_{x \mapsto 2^+} \frac{\sqrt{x^2-4}}{x-2} = \lim_{x \mapsto 2^+} \sqrt{\frac{(x-2)(x+2)}{(x-2)^2}} = \lim_{x \mapsto 2^+} \sqrt{\frac{x+2}{x-2}} = +\infty$$

$$\lim_{x \mapsto -2^-} \frac{\sqrt{x^2-4}}{x+2} = \lim_{x \mapsto 2^-} -\frac{\sqrt{x^2-4}}{-(x+2)} = \lim_{x \mapsto 2^-} -\sqrt{\frac{(x-2)(x+2)}{(x+2)^2}} = \lim_{x \mapsto 2^+} -\sqrt{\frac{x-2}{x+2}} = -\infty$$

$$\lim_{x \mapsto 1} \left(\frac{x^2 + x - 2}{x^3 + 4x^2 - 8x + 3} \right) = \lim_{x \mapsto 1} \frac{(x-1)(x+2)}{(x-1)(x^2 + 5x - 3)} = 1$$

$$\lim_{x \mapsto 1} \left(\sin\left(\frac{5}{1-x^3}\right)(x^2 - 2x + 1) \right) = \lim_{x \mapsto 1} \sin\left(\frac{5}{1-x^3}\right)(x-1)^2 = 0$$

لأن : $\forall x \in IR \quad \left| \sin\left(\frac{5}{1-x^3}\right) \right| \leq 1 \implies \forall x \in IR \quad \left| \sin\left(\frac{5}{1-x^3}\right)(x-1)^2 \right| \leq (x-1)^2$

و $\lim_{x \mapsto 1} (x-1)^2 = 0$ و ذلك حسب مصاديق تقارب نهاية دالة.

لأن: $\forall x > 0 \quad \cos\left(\frac{1}{x}\right) \geq +1 \implies \forall x > 0 \quad \frac{1}{x} + \cos\left(\frac{1}{x}\right) \geq \frac{1}{x} + 1$ و $\lim_{x \mapsto 0^+} \frac{1}{x} + 1 = +\infty$ $\lim_{x \mapsto 0^+} \frac{1}{x} + \cos\left(\frac{1}{x}\right) = +\infty$

وذلك حسب مصاديق تقارب نهاية دالة.

تمرين3 :

$$\lim_{x \mapsto 0}\left(\frac{1}{\sqrt{3}\cos x - \sin^2 x - \sqrt{3}}\right) = \lim_{x \mapsto 0}\left(\frac{1}{\sqrt{3}(\cos x - 1) - \sin^2 x}\right) = \lim_{x \mapsto 0}\left(\frac{-1}{\sqrt{3}(1 - \cos(x)) + \sin^2 x}\right)$$

$$= \lim_{x \mapsto 0}\left(\frac{-\dfrac{1}{x^2}}{\sqrt{3}\left(\dfrac{1 - \cos(x)}{x^2}\right) + \left(\dfrac{\sin x}{x}\right)^2}\right) = -\infty$$

$$\left(\;\lim_{x \mapsto 0} -\frac{1}{x^2} = -\infty \quad \text{و} \quad \lim_{x \mapsto 0} \sqrt{3}\left(\frac{1 - \cos(x)}{x^2}\right) + \left(\frac{\sin x}{x}\right)^2 = \frac{\sqrt{3}}{2} + 1 : \text{لأن}\;\right)$$

$$\lim_{x \mapsto 0}\left(\frac{\sqrt{3}\cos x - \sin^2 x - \sqrt{3}}{x^2}\right) = \lim_{x \mapsto 0}\left(-\sqrt{3}\left(\frac{1 - \cos(x)}{x^2}\right) - \left(\frac{\sin(x)}{x}\right)^2\right) = \frac{-\sqrt{3}}{2} - 1$$

$$\lim_{x \mapsto \frac{\pi}{6}}\left(\frac{\cos x - \sqrt{3}\sin x}{6x - \pi}\right) = \lim_{x \mapsto \frac{\pi}{6}}\left(\frac{2\left(\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x\right)}{6\left(x - \dfrac{\pi}{6}\right)}\right) = \lim_{x \mapsto \frac{\pi}{6}}\left(\frac{1}{3}\frac{\left(\sin\left(\dfrac{\pi}{6}\right)\cos x - \cos\left(\dfrac{\pi}{6}\right)\sin x\right)}{\left(x - \dfrac{\pi}{6}\right)}\right)$$

$$= \lim_{x \mapsto \frac{\pi}{6}}\left(\frac{1}{3}\frac{\sin\left(\dfrac{\pi}{6} - x\right)}{\left(x - \dfrac{\pi}{6}\right)}\right) == \lim_{x \mapsto \frac{\pi}{6}}\left(\frac{-1}{3}\frac{\sin\left(x - \dfrac{\pi}{6}\right)}{\left(x - \dfrac{\pi}{6}\right)}\right) = \lim_{t \mapsto 0}\left(\frac{-1}{3}\frac{\sin(t)}{t}\right) = \frac{-1}{3}$$

<div dir="rtl">

💬 (قمنا بتغيير المتغير x وذلك بوضع $t = x - \dfrac{\pi}{6}$)، كما يمكن إجراء تغيير المتغير منذ البداية.

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$$\lim_{x \mapsto \frac{\pi}{4}}\left(\frac{\tan x - 1}{2\cos x - \sqrt{2}}\right) = \lim_{x \mapsto \frac{\pi}{4}}\left(\frac{\tan x - \tan\left(\dfrac{\pi}{4}\right)}{2\cos x - \sqrt{2}}\right) = \lim_{x \mapsto \frac{\pi}{4}}\left(\frac{\left(1 + \tan(x)\tan\left(\dfrac{\pi}{4}\right)\right)\tan\left(x - \dfrac{\pi}{4}\right)}{2\cos x - \sqrt{2}}\right) \quad t = x - \frac{\pi}{4} \;\text{نضع}$$

$$\lim_{x \mapsto \frac{\pi}{4}}\left(\frac{\tan x - 1}{2\cos x - \sqrt{2}}\right) = \lim_{t \mapsto 0}\left(\frac{\left(1 + \tan\left(t + \dfrac{\pi}{4}\right)\right)\tan(t)}{2\cos\left(t + \dfrac{\pi}{4}\right) - \sqrt{2}}\right) = \lim_{t \mapsto 0}\left(\frac{\left(1 + \tan\left(t + \dfrac{\pi}{4}\right)\right)\tan(t)}{2\left(\cos(t)\dfrac{\sqrt{2}}{2} - \sin(t)\dfrac{\sqrt{2}}{2}\right) - \sqrt{2}}\right)$$

$$= \lim_{t \mapsto 0}\left(\frac{\left(1 + \tan\left(t + \dfrac{\pi}{4}\right)\right)\tan(t)}{-\sqrt{2}(1 - \cos(t)) - \sqrt{2}\sin(t)}\right) = \lim_{t \mapsto 0}\left(\frac{\left(1 + \tan\left(t + \dfrac{\pi}{4}\right)\right)\dfrac{\tan(t)}{t}}{-\sqrt{2}\,t\dfrac{(1 - \cos(t))}{t^2} - \sqrt{2}\dfrac{\sin(t)}{t}}\right)$$

$$= \frac{(1 + 1) \times 1}{-\sqrt{2} \times 0 \times \dfrac{1}{2} - \sqrt{2} \times 1} = -\sqrt{2}$$

<div dir="rtl">

💬 لاحظ أن استعمال الخاصية $\tan(a - b) = \dfrac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$

💬 بالشكل $\tan(a) - \tan(b) = \left(1 + \tan(a)\tan(b)\right)\tan(a - b)$ أفضل من استعمال تغيير المتغير من البداية.

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$$\ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sqrt{1-\cos(x)} - \sqrt{1-\sin(x)}}{1-\tan(x)} \right) = \ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sin(x) - \cos(x)}{(1+\tan(x))\tan\left(x-\frac{\pi}{4}\right)} \times \frac{1}{\sqrt{1-\cos(x)} + \sqrt{1-\sin(x)}} \right)$$

$$= \ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sqrt{2}\left(\sin(x)\cos\left(\frac{\pi}{4}\right) - \cos(x)\sin\left(\frac{\pi}{4}\right) \right)}{(1+\tan(x))\tan\left(x-\frac{\pi}{4}\right)} \times \frac{1}{\sqrt{1-\cos(x)} + \sqrt{1-\sin(x)}} \right)$$

$$= \ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sin\left(x-\frac{\pi}{4}\right)}{\tan\left(x-\frac{\pi}{4}\right)} \times \frac{\sqrt{2}}{(1+\tan(x))\left(\sqrt{1-\cos(x)} + \sqrt{1-\sin(x)}\right)} \right)$$

و بما أن :
$$\ell im_{x \mapsto \frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\tan\left(x-\frac{\pi}{4}\right)} = \ell im_{t \mapsto 0} \frac{\sin(t)}{\tan(t)} = \ell im_{t \mapsto 0} \frac{\frac{\sin(t)}{t}}{\frac{\tan(t)}{t}} = \frac{1}{1} = 1$$

و
$$\ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sqrt{2}}{(1+\tan(x))\left(\sqrt{1-\cos(x)} + \sqrt{1-\sin(x)}\right)} \right) = \frac{\sqrt{2}}{(1+1)\left(\sqrt{1-\frac{\sqrt{2}}{2}} + \sqrt{1-\frac{\sqrt{2}}{2}} \right)}$$

$$= \frac{\sqrt{2}}{4\sqrt{\frac{2-\sqrt{2}}{2}}} = \frac{\sqrt{4+2\sqrt{2}}}{4}$$

فإن :
$$\ell im_{x \mapsto \frac{\pi}{4}} \left(\frac{\sqrt{1-\cos(x)} - \sqrt{1-\sin(x)}}{1-\tan(x)} \right) = \frac{\sqrt{4+2\sqrt{2}}}{4}$$

تمرين4 : $f_a(x) = \dfrac{1}{x+a} - \dfrac{a^2 x^2}{x^3 + a^3}$

1) لدينا: $x \neq -a \Leftrightarrow \begin{cases} x \neq -a \\ x^3 \neq (-a)^3 \end{cases} \Leftrightarrow \begin{cases} x + a \neq 0 \\ x^3 + a^3 \neq 0 \end{cases} \Leftrightarrow x \in Df_a$ منه $\quad Df_a = IR_{\setminus\{-a\}}$

2) لدينا : $f_a(x) = \dfrac{1}{x+a} - \dfrac{a^2 x^2}{x^3 + a^3} = \dfrac{x^2 - ax + a^2}{x^3 + a^3} - \dfrac{a^2 x^2}{x^3 + a^3} = \dfrac{(1-a^2)x^2 - ax + a^2}{x^3 + a^3}$

نضع : $g_a(x) = (1-a^2)x^2 - ax + a^2$ ، إذن : $g_a(-a) = (1-a^2)a^2 + a^2 + a^2 = 3a^2 - a^4 = a^2(3 - a^2)$

- إذا كان : $a = 0$ فإن: $f_0(x) = \dfrac{x^2}{x^3} = \dfrac{1}{x}$ منه : $\underset{\substack{x \mapsto 0 \\ x>0}}{\ell im}\, f_0(x) = +\infty$ و $\underset{\substack{x \mapsto 0 \\ x<0}}{\ell im}\, f_0(x) = -\infty$

- إذا كان : $a^2 < 3$ و $a \neq 0$ (أي $a \in \left]-\sqrt{3};0\right[\cup \left]0;\sqrt{3}\right[$) فإن: $g_a(-a) > 0$
  منه : $\underset{\substack{x \mapsto -a \\ x>-a}}{\ell im}\, f_a(x) = +\infty$ و $\underset{\substack{x \mapsto -a \\ x<-a}}{\ell im}\, f_a(x) = -\infty$

- إذا كان : $a^2 > 3$ و (أي $a \in \left]-\infty;-\sqrt{3}\right[\cup \left]\sqrt{3};+\infty\right[$) فإن: $g_a(-a) < 0$
  منه : $\underset{\substack{x \mapsto -a \\ x>-a}}{\ell im}\, f_a(x) = -\infty$ و $\underset{\substack{x \mapsto -a \\ x<-a}}{\ell im}\, f_a(x) = +\infty$

- إذا كان : $a = \sqrt{3}$ فإن: $g_a(-\sqrt{3}) = 0$ ، منه :

$\underset{x \mapsto -\sqrt{3}}{\ell im}\, f_{\sqrt{3}}(x) = \underset{x \mapsto -\sqrt{3}}{\ell im}\, \dfrac{-2x^2 - \sqrt{3}x + 3}{x^3 + (\sqrt{3})^3} = \underset{x \mapsto -\sqrt{3}}{\ell im}\, \dfrac{(x+\sqrt{3})(\sqrt{3} - 2x)}{(x+\sqrt{3})(x^2 - x\sqrt{3} + 3)} = \underset{x \mapsto -\sqrt{3}}{\ell im}\, \dfrac{\sqrt{3} - 2x}{x^2 - x\sqrt{3} + 3} = \dfrac{3\sqrt{3}}{9} = \dfrac{\sqrt{3}}{3}$

- إذا كان : $a = -\sqrt{3}$ فإن: $g_a(\sqrt{3}) = 0$ ، منه :

$\underset{x \mapsto \sqrt{3}}{\ell im}\, f_{\sqrt{3}}(x) = \underset{x \mapsto \sqrt{3}}{\ell im}\, \dfrac{-2x^2 + \sqrt{3}x + 3}{x^3 - (\sqrt{3})^3} = \underset{x \mapsto \sqrt{3}}{\ell im}\, \dfrac{(x-\sqrt{3})(-\sqrt{3} - 2x)}{(x-\sqrt{3})(x^2 + x\sqrt{3} + 3)} = \underset{x \mapsto \sqrt{3}}{\ell im}\, \dfrac{-\sqrt{3} - 2x}{x^2 + x\sqrt{3} + 3} = \dfrac{-3\sqrt{3}}{9} = \dfrac{-\sqrt{3}}{3}$

خلاصة: القيم التي تجيب عن السؤال هي : $a = \sqrt{3}$ و $a = -\sqrt{3}$

🌿 استعملنا المحددة للتعميل

تمرين5 : $f(x)=\dfrac{mx^3+(m-2)x^2+(m-1)x+m-3}{x(x-2)(x-3)}$

1) لدينا: $Df=IR_{\{0;2;3\}}$ ، إذن : $x\in Df \Leftrightarrow x\neq 0$ et $x-2\neq 0$ et $x-3\neq 0$

2) ندرس النهاية في اللانهاية :

- إذاكان $m=0$: فإن $\displaystyle\lim_{x\mapsto+\infty} f(x)=\lim_{x\mapsto+\infty}\dfrac{-2x^2-x-3}{x(x-2)(x-3)}=\lim_{x\mapsto+\infty}\dfrac{-2x^2}{x^3}=\lim_{x\mapsto+\infty}\dfrac{-2}{x}=0$ و $\displaystyle\lim_{x\mapsto-\infty} f(x)=0$

- إذاكان $m\neq 0$: فإن $\displaystyle\lim_{x\mapsto+\infty} f(x)=\lim_{x\mapsto+\infty}\dfrac{mx^3}{x^3}=m$ و $\displaystyle\lim_{x\mapsto-\infty} f(x)=m$

يمكن أن نلخص الحالات في النتيجة التالية، $\displaystyle\lim_{x\mapsto+\infty} f(x)=\lim_{x\mapsto-\infty} f(x)=m$ $\forall m\in IR$

نضع: $g(x)=mx^3+(m-2)x^2+(m-1)x+m-3$

$g(0)=m-3$ و $g(2)=8m+4(m-2)+2(m-1)+m-3=15m-13$

و $g(3)=27m+9(m-2)+3(m-1)+m-3=40m-24$

ندرس النهاية في 0 :

- إذاكان $m=3$: فإن $\displaystyle\lim_{x\mapsto 0} f(x)=\lim_{x\mapsto 0}\dfrac{3x^3+x^2+2x}{x(x-2)(x-3)}=\lim_{x\mapsto 0}\dfrac{3x^2+x+2}{(x-2)(x-3)}=\dfrac{2}{6}=\dfrac{1}{3}$

- إذاكان $m>3$: فإن $\displaystyle\lim_{\substack{x\mapsto 0\\x<0}} f(x)=+\infty$ $\left(\dfrac{\ell>0}{0^+}\right)$ و $\displaystyle\lim_{\substack{x\mapsto 0\\x>0}} f(x)=-\infty$ $\left(\dfrac{\ell>0}{0^-}\right)$

- إذاكان $m<3$: فإن $\displaystyle\lim_{\substack{x\mapsto 0\\x<0}} f(x)=-\infty$ $\left(\dfrac{\ell<0}{0^+}\right)$ و $\displaystyle\lim_{\substack{x\mapsto 0\\x>0}} f(x)=+\infty$ $\left(\dfrac{\ell<0}{0^-}\right)$

تمرين6 : $f(x)=\dfrac{x^2\sqrt{x+2}-8}{4-x^2}$

1) لدينا: $x\in Df \Leftrightarrow \begin{cases} 4-x^2\neq 0\\ x+2\geq 0\end{cases} \Leftrightarrow \begin{cases} x\neq 2 \text{ et } x\neq -2\\ x\geq -2\end{cases} \Leftrightarrow \begin{cases} x\neq 2\\ x>-2\end{cases} \Leftrightarrow x\in]-2;2[\cup]2;+\infty[$

إذن : $Df=]-2;2[\cup]2;+\infty[$

2) إذاكان $-\infty$: $\displaystyle\lim_{x\mapsto+\infty} f(x)=\lim_{x\mapsto+\infty}\dfrac{x^2\sqrt{x+2}-8}{4-x^2}=\lim_{x\mapsto+\infty}\dfrac{x^2\left(\sqrt{x+2}-\dfrac{8}{x^2}\right)}{x^2\left(\dfrac{4}{x^2}-1\right)}=\lim_{x\mapsto+\infty}\dfrac{\sqrt{x+2}-\dfrac{8}{x^2}}{\dfrac{4}{x^2}-1}=-\infty$

3) $\displaystyle\lim_{x\mapsto 2} f(x)=\lim_{x\mapsto 2}\dfrac{x^2\sqrt{x+2}-8}{4-x^2}=\lim_{x\mapsto 2}\dfrac{x^2\left(\sqrt{x+2}-2\right)+2x^2-8}{4-x^2}=\lim_{x\mapsto 2}\dfrac{x^2\dfrac{x+2-4}{\sqrt{x+2}+2}+2(x^2-4)}{4-x^2}$

$=\displaystyle\lim_{x\mapsto 2}\dfrac{x^2\dfrac{x-2}{\sqrt{x+2}+2}}{4-x^2}-2=\lim_{x\mapsto 2}\dfrac{-x^2}{(x+2)(\sqrt{x+2}+2)}-2=\dfrac{-4}{16}-2=\dfrac{-9}{4}$

إذن f تقبل تمديدا بالاتصال في 2 تمديده هو: $\begin{cases} f(x)=\dfrac{x^2\sqrt{x+2}-8}{4-x^2}; \ x\in]-2;2[\cup]2;+\infty[\\ f(2)=\dfrac{9}{4}\end{cases}$